a study on
Robust Mechanisms for Social Coordination

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CDC Workshop: Distributed Autonomy and Human-Machine Networks
December 14, 2015
Central Goal

Derive efficient system-wide behavior through the design of admissible control algorithms.
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Derive **efficient** system-wide behavior through the design of **admissible** control algorithms

**efficient**: desirable allocation for any network demands

**admissible**: local/aggregate incentive mechanism

**challenge**: uncontrollable entities, unknown sensitivity
Multiagent coordination

Central Goal

Derive efficient system-wide behavior through the design of admissible control algorithms

Are there robust mechanisms for coordinating social behavior?

(guarantees irrespective of populations’ demands/sensitivities)
Motivation:

- Uninfluenced systems often exhibit poor system behavior

\[ c(x) = x \]

\[ c(x) = 1 \]

unit flow of traffic

optimal outcome vs. self-interested outcome
Motivation:

- Uninfluenced systems often exhibit poor system behavior

Case study: Pigou’s network

$\frac{1}{2}$

$\frac{1}{2}$

$c(x) = x$

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Unit flow of traffic

Optimal outcome vs. self-interested outcome

3/4
Motivation:
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3/4 1
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![Network Diagram]

$c(x) = x$

$c(x) = 1$

Unit flow of traffic

Optimal outcome vs. self-interested outcome

Self-interested outcome 33% worse than optimal outcome

3/4 1
Case study: *Braess’ paradox*

**Motivation:**
- Uninfluenced systems often exhibit poor system behavior
- Natural influencing mechanisms need not lead to intuitive outcomes
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- Uninfluenced systems often exhibit poor system behavior
- Natural influencing mechanisms need not lead to intuitive outcomes

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c(x) = x \quad c(x) = 1
\]

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c(x) = x \quad c(x) = 1 \quad c(x) = 0
\]

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c(x) = 1 \quad c(x) = x
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original network vs. original network + extra edge
Motivation:
- Uninfluenced systems often exhibit poor system behavior
- Natural influencing mechanisms need not lead to intuitive outcomes

Case study: Braess’ paradox

Original network vs. original network + extra edge

1.5
Motivation:
- Uninfluenced systems often exhibit poor system behavior
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**Case study: Braess’ paradox**

\[
c(x) = x \quad \text{vs.} \quad c(x) = 1
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original network vs. original network + extra edge

1.5 vs. 2
Motivation:
- Uninfluenced systems often exhibit poor system behavior
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original network vs. original network + extra edge

\[ c(x) = x \]
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\[ \frac{1}{2} \]
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Motivation:

Case study: Braess’ paradox

additional resources resulted in 33% worse system performance
Can taxes be employed to help coordinate behavior?

**Structure of Mechanism**
- path based vs. edge based?
- anonymous vs. discriminatory?
- fixed vs. behavior dependent?

**Informational Demand**
- network demands?
- network structure?
- population sensitivity?
Can taxes be employed to help coordinate behavior?

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- network demands? None.
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Recall: Pigou’s network

Can taxes be employed to help coordinate behavior?
(discourage users from using top edge)

\[ c(x) = x \]

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Nash flow

1
0
1

33% degradation in performance

Optimal flow

1/2
1/2
3/4
Can taxes be employed to help coordinate behavior?
(discourage users from using top edge)

\[ c(x) = x \quad \quad \quad \quad \quad t(x) = x \]

\[ c(x) = 1 \]
Can taxes be employed to help coordinate behavior?
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\[ c(x) = 2x \]
\[ t(x) = x \]

Recall: Pigou’s network

\[ c(x) = 1 \]
Can taxes be employed to help coordinate behavior?
(discourage users from using top edge)

$c(x) = x$
$t(x) = x$

$c(x) = 2x$

Optimal flow

$3/4$

$1/2$

$1/2$
Recall: Pigou’s network

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Nash flow

Optimal flow
Can taxes be employed to help coordinate behavior? (discourage users from using top edge)

\[ c(x) = x \]

\[ t(x) = x \]

Nash flow

Optimal flow

tolls are an effective remedy for efficiency loss

Recall: Pigou’s network
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Can taxes be employed to help coordinate behavior?
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\[ c(x) = x \]
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\[ c(x) = x, \quad c(x) = 2x, \quad t(x) = x \]

\[ c(x) = 1 \]

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- path based vs. edge based? ✓
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Model: Users’ sensitivities between congestion and tolls unknown

$$\textbf{Cost} \ (\text{agent } i) = \textbf{QoS} + \beta_i \cdot \textbf{Incentives}$$

- (congestion)
- (tolls)

(unknown/heterogeneous sensitivity)

$$\beta : [0, 1] \rightarrow [\beta_{\min}, \beta_{\max}]$$

**Model**: Users' sensitivities between congestion and tolls unknown

\[
\text{Cost} (\text{agent } i) = \text{QoS} + \beta_i \cdot \text{Incentives}
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\[
(\text{congestion}) \quad \quad \quad \quad \quad \quad (\text{tolls})
\]

\[
(\text{unknown/heterogenous sensitivity})
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\[
\beta : [0, 1] \rightarrow [\beta^\min, \beta^\max]
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**Nash flow**: A flow such that no agent has a unilateral incentive to deviate.

Model: Users’ sensitivities between congestion and tolls unknown

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Nash flow: A flow such that no agent has a unilateral incentive to deviate.

structure of mechanism informational demands \hspace{2cm} \rightarrow \hspace{2cm} \text{efficiency guarantees?}

Performance

taxation mechanism

fixed
flow dependent

information demands

network structure
network demands
users’ sensitivities
best efficiency guarantees

?
Example

previous example

\[ c(x) = 2x \]

\[ t(x) = x \]

\[ c(x) = 1 \]
Pigovian taxes

Pigovian Tax

\[ t_e(f_e) = f_e \cdot c'_e(f_e) \]

### Fixed taxes

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### Fixed Tax

\[ t_e(f_e) = c_e \]

---

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### Our contributions

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(Pigovian tax)  

\[ c_e(f_e) + t_e(f_e) \rightarrow c_e(f_e) + f_e \cdot c'_e(f_e) \]  

100% efficiency guarantees

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(Pigovian tax) \[ c_e(f_e) + t_e(f_e) \rightarrow c_e(f_e) + f_e \cdot c'_e(f_e) \rightarrow 100\% \text{ efficiency guarantees} \]

(Fixed taxes) \[ \checkmark \]

(Brown & JRM) \[ \checkmark \]

(Brown & JRM) \[ \checkmark \]

(Pigovian tax) \[ \checkmark \]

(our setting) \[ c_e(f_e) + \beta_x t_e(f_e) \]

\[ \beta_x \in [\beta^{\min}, \beta^{\max}] \]

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\[
\begin{align*}
    (\text{Pigovian tax}) & : c_e(f_e) + t_e(f_e) \\
    (\text{our setting}) & : c_e(f_e) + \beta x t_e(f_e) \\
\end{align*}
\]

\[
\begin{align*}
    & c_e(f_e) + f_e \cdot c'_e(f_e) \\
\end{align*}
\]

(Brown and JRM, “Optimal Mechanisms for Robust Coordination in Congestion Games,” 2015.)
Our contributions

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(Pigovian tax) \[ c_e(f_e) + t_e(f_e) \]

(Fixed taxes) \[ t_e(f_e) = \lim_{k \to \infty} k(c_e(f_e) + f_e \cdot c'_e(f_e)) \]

(our setting) \[ c_e(f_e) + \beta_x t_e(f_e) \]

\[ \beta_x \in [\beta_{\min}, \beta_{\max}] \]

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(Pigovian tax) $c_e(f_e) + t_e(f_e)$

(our setting) $c_e(f_e) + \beta_x t_e(f_e)$

$\beta_x \in [\beta_{\min}, \beta_{\max}]$

$t_e(f_e) = \lim_{k \to \infty} k(c_e(f_e) + f_e \cdot c'_e(f_e))$

(Requires large tolls)

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What if we have constraints on the maximum toll?

Our contributions

Setting: Parallel networks, affine latency functions

\[ c_e(f_e) = a_e \cdot f_e + b_e \]

Theorem: Let \( B \) be the maximum toll. Tolls that optimize worst-case efficiency are

\[ t_e(f_e) = k_1(\beta_{\text{min}}, \beta_{\text{max}}, B) f_e + k_2(\beta_{\text{min}}, \beta_{\text{max}}, B) \]

Discriminatory tolls

Previous setting: Anonymous tolls (every driver sees same price)

Question: Could we exploit discriminatory pricing to improve efficiency guarantees?

Discriminatory tolls

**Previous setting:** Anonymous tolls (every driver sees same price)

**Question:** Could we exploit discriminatory pricing to improve efficiency guarantees?

\[
\text{Cost (agent } i \text{)} = \text{QoS} + \beta_i \cdot \text{Incentives}
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(congestion) (tolls)

(unknown/heterogenous sensitivity)

\[
\beta : [0, 1] \rightarrow [\beta_{\text{min}}, \beta_{\text{max}}]
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\(\beta : [0, 1] \rightarrow [\beta_{\text{min}}, \beta_{\text{max}}]\)

**Approach:** Discriminate according to price sensitivity

\(\beta_{\text{min}} \leq \beta \leq \beta_{\text{max}}\)

Discriminatory tolls

Previous setting: Anonymous tolls (every driver sees same price)

Question: Could we exploit discriminatory pricing to improve efficiency guarantees?

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Approach: Discriminate according to price sensitivity

Discriminatory tolls

Previous setting: Anonymous tolls (every driver sees same price)

Questions:

- How should you discriminate?
- What are the optimal taxes within each bin?
- What are the optimal bin boundaries?
- How finely should we discriminate? (errors?)
- Comparison between discriminatory and non-discriminatory mechanisms?
- Adaptive tolling mechanisms?

Approach: Discriminate according to price sensitivity

A lemma

Existing design:

\[ \text{PoA}(t, \beta_{\text{min}}, \beta_{\text{max}}) \]

\[ t_e(\cdot) \]

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\[ \text{PoA}(t, \beta_{\text{min}}, \beta_{\text{max}}) \]

\[ \beta_{\text{min}} \quad \downarrow \quad \beta_{\text{max}} \]

\[ t_e(\cdot) \]

New specifications:

\[ \text{PoA}(\tilde{t}, \tilde{\beta}_{\text{min}}, \tilde{\beta}_{\text{max}}) \]

\[ \tilde{\beta}_{\text{min}} \quad \downarrow \quad \tilde{\beta}_{\text{max}} \]

\[ \tilde{t}_e(\cdot) \]

Can we exploit the original design/analysis for new domain?

A lemma

Existing design:

PoA\( (t, \beta^{\min}, \beta^{\max}) \)

\( \beta^{\min} \) \( \xrightarrow{t_e(\cdot)} \) \( \beta^{\max} \)

scale of uncertainty
driving factor of PoA

\( \frac{\beta^{\max}}{\beta^{\min}} \)

New specifications:

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\( \tilde{\beta}^{\min} \) \( \xrightarrow{\tilde{t}_e(\cdot)} \) \( \tilde{\beta}^{\max} \)

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Existing design:

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\[ \beta_{\text{min}} \rightarrow t_e(\cdot) \rightarrow \beta_{\text{max}} \]

New specifications:

PoA\( (\tilde{t}, \tilde{\beta}_{\text{min}}, \tilde{\beta}_{\text{max}}) \)

\[ \tilde{\beta}_{\text{min}} \rightarrow \left( \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \right) \tilde{\beta}_{\text{min}} \rightarrow \left( \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \right)^2 \tilde{\beta}_{\text{min}} \rightarrow \tilde{\beta}_{\text{max}} \]

scale of uncertainty
driving factor of PoA

\[ \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \]

Existing design:

\[
\text{PoA}(t, \beta_{\min}, \beta_{\max})
\]

\[
\beta_{\min} \quad \downarrow \\
\tilde{t}_e(\cdot) \\
\beta_{\max}
\]

New specifications:

\[
\text{PoA}(\tilde{t}, \tilde{\beta}_{\min}, \tilde{\beta}_{\max})
\]

\[
\tilde{\beta}_{\min} \quad \downarrow \\
\tilde{t}_e^1(\cdot) = \left( \frac{\beta_{\min}}{\tilde{\beta}_{\min}} \right) t_e(\cdot) \\
\tilde{\beta}_{\max}
\]

scale of uncertainty driving factor of PoA

\[
\frac{\beta_{\max}}{\beta_{\min}}
\]

scale tax by normalizing left boundary

A lemma

Existing design:

\[
\text{PoA}(t, \beta_{\min}, \beta_{\max})
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\[
\beta_{\min} \downarrow t_e(\cdot) \downarrow \beta_{\max}
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New specifications:

\[
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\tilde{\beta}_{\min} \downarrow \left( \frac{\beta_{\max}}{\beta_{\min}} \right) \tilde{\beta}_{\min} \downarrow \tilde{t}_e(\cdot) = \left( \frac{\beta_{\min}}{\tilde{\beta}_{\min}} \right) \left( \frac{\beta_{\min}}{\beta_{\max}} \right) t_e(\cdot)
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driving factor of PoA

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\[ \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \]

New specifications:

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\[ \tilde{\beta}_{\text{min}} \rightarrow \tilde{\beta}_{\text{max}} \]

\[ \tilde{t}_e^3(\cdot) = \left( \frac{\beta_{\text{min}}}{\tilde{\beta}_{\text{min}}} \right) \left( \frac{\beta_{\text{min}}}{\beta_{\text{max}}} \right)^2 t_e(\cdot) \]

A lemma

Existing design:

\[ \text{PoA}(t, \beta_{\text{min}}, \beta_{\text{max}}) \]

\[ \beta_{\text{min}} \quad t_e(\cdot) \quad \beta_{\text{max}} \]

New specifications:

\[ \text{PoA}(\tilde{t}, \tilde{\beta}_{\text{min}}, \tilde{\beta}_{\text{max}}) \]

\[ \tilde{\beta}_{\text{min}} \quad \tilde{t}_e(\cdot) \quad \tilde{\beta}_{\text{max}} \]

Lemma: \[ \text{PoA}(\tilde{t}, \tilde{\beta}_{\text{min}}, \tilde{\beta}_{\text{max}}) \leq \text{PoA}(t, \beta_{\text{min}}, \beta_{\text{max}}) \]

(irrespective of network characteristics)

What is the best discriminatory taxation mechanism to use?

\[ \beta_{\text{min}} \leq \beta \leq \beta_{\text{max}} \]
Discriminatory taxes

What is the best discriminatory taxation mechanism to use?

$\beta_{\text{min}} \quad \text{scale of uncertainty} \quad \beta_{\text{max}}$

efficiency of resulting equilibria

What is the best discriminatory taxation mechanism to use?

Approach: Discriminate to minimize maximum scale of uncertainty

What is the best discriminatory taxation mechanism to use?

all bins have the same scale of uncertainty

What is the best discriminatory taxation mechanism to use?

\[ \min \beta_{min} \quad P_1 \quad \min \beta_{max} \quad P_2 \quad \cdots \quad \max \beta_{max} \quad P_m \]

\[ t_e^1(\cdot) \quad t_e^2(\cdot) \quad t_e^m(\cdot) \]

What is the best discriminatory taxation mechanism to use?

Recall Previous Theorem
Let $B$ be the maximum toll. Tolls that optimize worst-case efficiency are

$$t_e(f_e) = k_1(\beta^{\min}, \beta^{\max}, B)f_e + k_2(\beta^{\min}, \beta^{\max}, B)$$

(lower boundary) (parallel affine networks) (upper boundary)
What is the best discriminatory taxation mechanism to use?

Recall Previous Theorem

Let $B$ be the maximum toll. Tolls that optimize worst-case efficiency are

$$t_e(f_e) = k_1(\beta_{\text{min}}, \beta_{\text{max}}, B)f_e + k_2(\beta_{\text{min}}, \beta_{\text{max}}, B)$$

Theorem: $\text{PoA}(m, B, \beta_{\text{min}}, \beta_{\text{max}})$

- decreasing in $m$
- goes to 1 if $B$ sufficiently large

Discriminatory taxes

What is the best discriminatory taxation mechanism to use?

\[ \beta_{\text{min}} \quad t^1_e(\cdot) \quad t^2_e(\cdot) \quad t^m_e(\cdot) \quad \beta_{\text{max}} \]

\[ \min \quad \max \]

\[ P_1 \quad P_2 \quad \cdots \quad P_m \]

\[ t_2 \quad e (\cdot) \quad t_1 \quad e (\cdot) \quad t_m \quad e (\cdot) \]

\[ \beta_{\text{max}} = \frac{\beta_{\text{max}}}{\beta_{\text{min}}} = \text{fixed} \]

Increasing maximum toll

Efficiency guarantees

What is the best discriminatory taxation mechanism to use?

$$\beta_{\text{min}} \leq t_e(\cdot) \leq t_e(\cdot) \leq \beta_{\text{max}}$$

GOAL
improve efficiency resulting equilibria

non-discriminatory tolls
(requires large tolls)

discriminatory tolls
(requires smaller tolls)

discrimination can compensate for magnitude of tolls

Discriminatory taxes

What is the best discriminatory taxation mechanism to use?

\[ \begin{align*}
\mathcal{P}_1 & \quad \mathcal{P}_2 & \quad \cdots & \quad \mathcal{P}_m \\
\beta_{\text{min}} & \quad t_e^1(\cdot) & \quad t_e^2(\cdot) & \quad t_e^m(\cdot) & \quad \beta_{\text{max}}
\end{align*} \]

**GOAL**

improve efficiency resulting equilibria

non-discriminatory tolls (requires large tolls)

discriminatory tolls (requires smaller tolls)

discrimination can compensate for magnitude of tolls

**Open Question**

What is the optimal discriminatory toll?

**Central Goal**

Derive **efficient** system-wide behavior through the design of **admissible** control algorithms.
Central Goal

Derive efficient system-wide behavior through the design of admissible control algorithms

challenges

Is there a general theory for robust social coordination?
What is the role of discrimination in multiagent coordination?
What about learning and influencing at the same time?
Game theoretic methods for distributed control:


Trade-offs in multiagent coordination: